TEXING 2-CELLS IN DERIVATORS

IAN COLEY

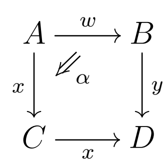
As of the writing of this document, there does not seem a unified way of TEXing commutative diagrams which include 2-cells. All seem to use the **xymatrix** package, but the implementation differs. To compile the TEX to follow, the reader should include

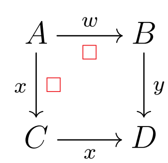
```
\usepackage[all, 2cell]{xy}
\UseAllTwocells
```

in the preamble of his or her TEX document.

The issue is in the all-important natural transformation. For instance, the method used by [MR14] is the following:

```
\xymatrix{
A \ar[r]^{w}_-{\;}="a" \ar[d]_{x}^-{\;}="b"&
  B \ar[d]^{y}\\
C \ar[r]_{x}&
  D \ar@{=>}"a";"b"^{\alpha} }
```

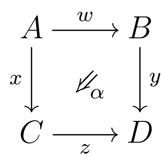


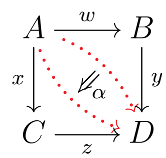


This involves only the standard **xymatrix** package. It creates two empty labels `\;` at the indicated boxes, and draws a labelled natural transformation arrow $\Rightarrow$ between them. The boxes are placed at the centre of the arrows by means of `-` written before the label name.

Another tactic, by [Gro16], gives:

```
\xymatrix{
A \ar[r]^{w} \ar[d]_{x} \drtwocell\omit{\alpha}&
  B \ar[d]^{y}\\
C \ar[r]_{z}&
  D }
```

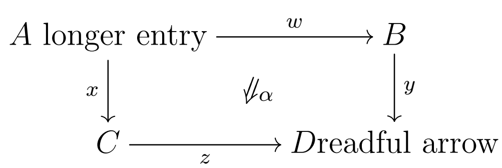


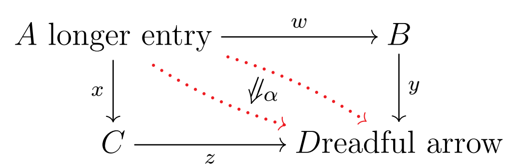


This uses the `2cell` option of `xymatrix`. It draws an invisible 2-cell between the upper-left and the bottom-right, but leaves the natural transformation associated to it.

This approach is a bit more technically involved, but places the natural transformation in the centre of the commutative diagram – a desirable location. However, it is harder to control the direction of the arrow, and in asymmetric diagrams the resulting picture suffers for it. The dotted arrows below help demonstrate why.[1]

```
\xymatrix{
A\text{ longer entry} \ar[r]^-{w} \ar[d]_{x} \drtwocell\omit{\alpha}&
  B \ar[d]^{y}\\
C \ar[r]_-{z}&
  D\text{readful arrow} }
```

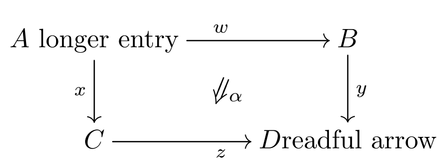




The 2-cell arrow is drawn to be along the perpendicular bisector of the straight arrow between the upper-left and bottom-right. As our square becomes more a rectangle, this direction less and less resembles the direction of the opposing diagonal. In pastings, this becomes awkward:

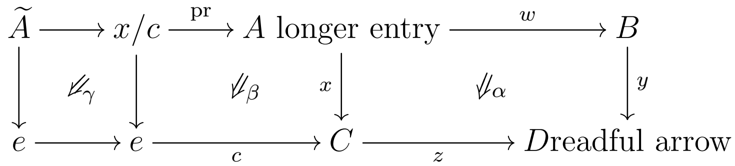
```
\xymatrix{
\widetilde A \ar[r] \ar[d] \drtwocell\omit{\gamma}&
  x/c \ar[r]^-{\text{pr}} \ar[d] \drtwocell\omit{\beta}&
  A\text{ longer entry} \ar[r]^-w \ar[d]_{x} \drtwocell\omit{\alpha}&
  B \ar[d]^{y}\\
e \ar[r]&
```

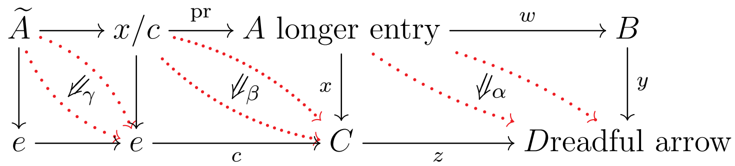
[1]As a side note: while it is not always necessary to use `-` for centring the labels on the outside square, in the case of asymmetric diagrams it is essential. Here is that diagram without centring the labels:



I do not prefer this style.

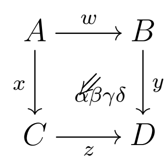
```
  e \ar[r]_-c&
  C \ar[r]_-z&
  D\text{readful arrow} }
```





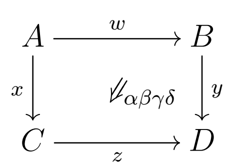
The arrow seems to change directions slowly. The labelling for the natural transformation seems also to be necessarily below $\Rightarrow$, and long labels become somewhat awkward:

```
\xymatrix{
A \ar[r]^{w} \ar[d]_{x} \drtwocell\omit{\alpha\beta\gamma\delta}&
  B \ar[d]^{y}\\
C \ar[r]_{z}&
  D }
```



This can be solved by adding whitespace within the `\omit{ }` and by lengthening the size of the commutative diagram, and it looks pretty good at the cost of losing the square shape and strictly diagonal arrow:

```
\xymatrix@C=4em{
A \ar[r]^{w} \ar[d]_{x} \drtwocell\omit{\quad\;\alpha\beta\gamma\delta}&
  B \ar[d]^{y}\\
C \ar[r]_{z}&
  D }
```

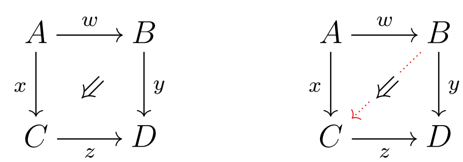


The method I propose attempts to address these problems. First, it puts an arrow pointing in a fixed cardinal direction, even when our commutative diagram is rectangular. Second, labelling may be done flexibly – it is not as built-in as Groth's approach, but it allows for longer arrows without the need for tweaking the diagram shape and adding whitespace. It relies on four objects built out of the standard $\Rightarrow$:

```
\newcommand{\swtrans}
  {\mathbin{\rotatebox[origin=c]{225}{$\Rightarrow$}}}
```

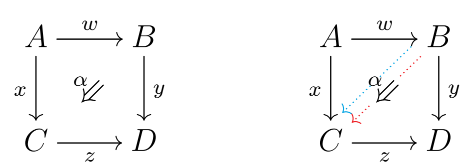
I call $\Swarrow$ the 'southwest transformation' (though Australian mathematicians may disagree with this terminology) based on its cardinal direction. A similar command can give one a southeast arrow, etc. We place this in the middle of our standard square as an in-line label of an invisible arrow:

```
\xymatrix{
A \ar[r]^{w} \ar[d]_{x}&
  B \ar[d]^{y} \ar@{}[dl]|\swtrans\\
C \ar[r]_{z}&
  D }
```



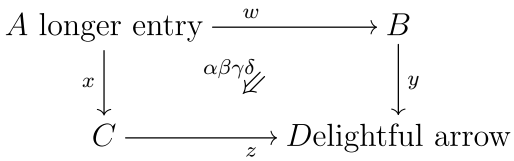
To label this arrow, we add another invisible arrow shifted slightly above or below, and put our label in-line as well:

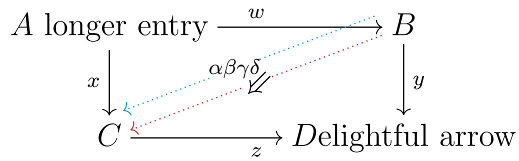
```
\xymatrix{
A \ar[r]^{w} \ar[d]_{x}&
  B \ar[d]^{y} \ar@{}[dl]|\swtrans \ar@{}[dl]<-1.0ex>|\alpha\\
C \ar[r]_{z}&
  D }
```



The key `<-1.0ex>` tells the arrow to move right (that is, negative left) by one length of the letter x (roughly). This may need to be tweaked depending on the size of your label and diagram. To repeat the example above,

```
\xymatrix{
A\text{ longer entry}\ar[r]^{w} \ar[d]_{x}&
  B \ar[d]^{y} \ar@{}[dl]|\swtrans
    \ar@{}[dl]<-1.5ex>|(0.55){\alpha\beta\gamma\delta}\\
C \ar[r]_{z}&
  D\text{elightful arrow} }
```

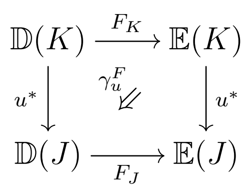


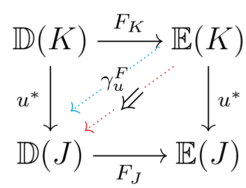


We have moved our cerulean label arrow by `<-1.5ex>|(0.55)`. The first means that we have shifted over a little further than in our first example, and the `(0.55)` means that we move our label from the dead middle of the arrow (which would be `(0.5)`) a little further towards the tip.

Granted, this is a little more labour-intensive than the previous two methods, but it means you are only changing two numbers. Here is another example, from the definition of a morphism of prederivators:

```
\newcommand{\D}{\mathbb D}
\newcommand{\E}{\mathbb E}
\xymatrix{
\D(K) \ar[r]^{F_K} \ar[d]_{u^\ast}&
  \E(K) \ar[d]^{u^\ast} \ar@{}[dl]|{\swtrans}
    \ar@{}@<-1.75ex>[dl]|{\gamma_u^F}\\
\D(J) \ar[r]_{F_J}&
  \E(J) }
```





In this case, we shift the arrow by 1.75 x-lengths and do not need to move it along the length of the arrow.

References

[Gro16] Moritz Groth. Revisiting the canonicity of canonical triangulations. arXiv:1602.04846, 2016.

[MR14] Fernando Muro and George Raptis. K-theory of derivators revisisted. arXiv:1402.1871, 2014.